# MHD Generator Characteristics with Insulator Wall Losses

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**The paper presents an analysis that delineates the important electrical effects of boundary layers on the insulator wall of a magnetohydrodynamic generator. For specified variation of the fluid properties associated with the boundary layer, the electrical characteristics may be used to identify shorting through the insulator wall boundary layer in an experimental investigation. Two numerical examples considering seeded gas MHD generators with equilibrium and nonequilibrium ionization show that if the channel dimension in the magnetic field direction is 10–100 times larger than the boundary-layer thickness, insulator wall shorting should have a negligible effect on the generator's performance. The equations are also generalized to investigate the effects of an insulator wall with finite scalar resistivity.**

## 1. Introduction

MAGNETOHYDRODYNAMIC generators designed to achieve large conductivities by nonequilibrium ionization have failed experimentally to produce the power levels predicted by theory.[1,2] Some mechanisms that could affect the generator's performance have been identified,[3] although the list may not be complete. The present work is a continuation of an investigation[4] exploring the losses introduced by the presence of boundary layers on the insulator walls of a nonequilibrium MHD generator. The purpose of the investigation is to gain an understanding of the boundary-layer effects so that they may be identified in an experimental situation. Further, it is hoped that the theory will suggest means of minimizing detrimental effects, by specifying a minimum generator size, for example.

The primary manifestations of the boundary layer in compressible flow are the velocity defect and the temperature increase near the wall. The result of the velocity defect is to deny the boundary layer an induced electric field to balance that arising from the current flow in the freestream. Since the electrodes are constant potential surfaces, the transverse electric field, $E_y$, is the same for both freestream and boundary layer. The induced electric field ($\bar{U} \times \bar{B}$) is larger than $E_y$ in the freestream and smaller in the boundary layer because the flow velocity is low. The current density, being the difference between these two electric fields, circulates within the channel and the load is consequently partially short circuited. A similar partial shorting by the boundary layer occurs in the Hall direction. By reducing the Hall electric field, $E_x$, the possibility of electron temperature elevation is seriously affected.

## 2. Analysis

By making certain assumptions about the state of the plasma, one can deduce the variations of transverse and Hall voltages with load current, when boundary-layer effects are important. In the following we assume 1) the freestream is uniform, 2) end effects are small, and 3) electric fields are uniform.

Figure 1 shows the coordinate system used, the directions of the current and gas flows, and of the applied magnetic field. The currents flowing in the freestream and returning through the boundary layers are also indicated. It is assumed that the effects of the turning current are small. This assumption is well justified when the channel's characteristic dimension in the $z$ direction is much less than that in the $y$ direction. The boundary-layer thickness $\delta_m$ is the larger of the thermal or viscous boundary-layer thicknesses.

Let $J$ be the transverse current density out of the generator to the load, $J_H$ the Hall current density out of the generator to the Hall load, and let primes denote quantities in the boundary layer that vary with $z$. $u$ is the varying velocity in the boundary layer.

Ohm's law in freestream can be written

$$j_x = \frac{\sigma}{1+\beta^2}[E_x - \beta(E_y - UB)]$$
$$j_y = \frac{\sigma}{1+\beta^2}[E_y - UB + \beta E_x] \quad (1)$$

and in boundary layer it is

$$j'_x = [\sigma'/(1+\beta'^2)][E_x - \beta'(E_y - uB)]$$
$$j'_y = [\sigma'/(1+\beta'^2)][E_y - uB + \beta' E_x] \quad (2)$$

The electric fields $E_x$ and $E_y$ are equal to those in the freestream by our assumption. Note that in the special case of a generator $j_y$ is negative and $j'_y$ is positive.

The conductivities of the boundary-layer and the freestream regions cannot be assumed known if the conductivity is

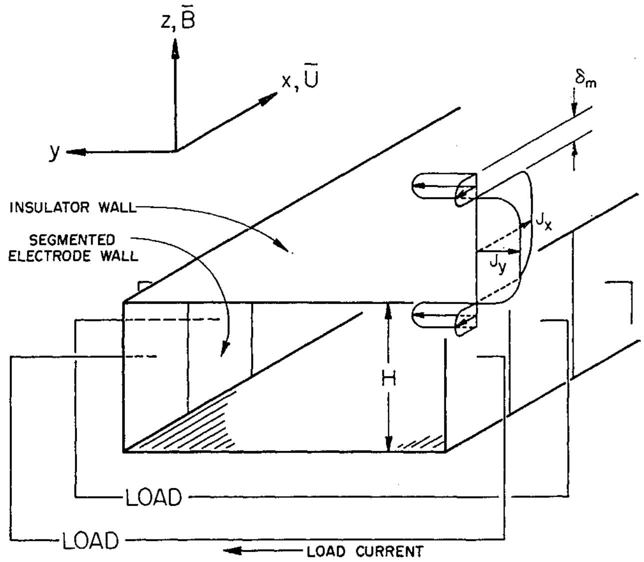


**Fig. 1 Generator schematic and coordinate system.**

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achieved by nonequilibrium ionization. This is the result of the nonlinear two-temperature conduction law. A relationship between these unknown conductivities must be assumed in the solution of this problem. Once the magnitudes of the electric fields in the plasma are obtained, the conductivities may then be calculated if the appropriate plasma properties are specified.

Conservation of current is required in the two directions. In the transverse direction

$$\int_0^{\delta_m} j'_y dz + \int_{\delta_m}^{H/2} j_y dz = J \frac{H}{2} \tag{3a}$$

and in the Hall direction

$$\int_0^{\delta_m} j'_x dz + \int_{\delta_m}^{H/2} j_x dz = J_H \frac{H}{2} \tag{3b}$$

Substituting Eqs. (2) into (3) and defining a quantity $\eta_n = 1 + 2(\delta_m/H)(f_n - 1)$ where $n = 1, 2, 3, 4$ and

$$\begin{aligned} f_1 &= \int_0^1 \frac{\sigma'}{\sigma} \frac{1+\beta^2}{1+\beta'^2} d\left(\frac{z}{\delta_m}\right) \\ f_2 &= \int_0^1 \frac{\sigma'}{\sigma} \frac{\beta'}{\beta} \frac{1+\beta^2}{1+\beta'^2} d\left(\frac{z}{\delta_m}\right) \\ f_3 &= \int_0^1 \frac{\sigma'}{\sigma} \frac{u}{U} \frac{\beta'}{\beta} \frac{1+\beta^2}{1+\beta'^2} d\left(\frac{z}{\delta_m}\right) \\ f_4 &= \int_0^1 \frac{\sigma'}{\sigma} \frac{u}{U} \frac{1+\beta^2}{1+\beta'^2} d\left(\frac{z}{\delta_m}\right) \end{aligned} \tag{4}$$

We have for the voltage-current characteristic and the Hall field

$$\begin{aligned} E_y \frac{\eta_1^2 + \eta_2^2\beta^2}{1+\beta^2} - UB \frac{\eta_1\eta_4 + \eta_2\eta_3\beta^2}{1+\beta^2} &= \eta_1 \frac{J}{\sigma} - \eta_2\beta \frac{J_H}{\sigma} \\ E_x = \frac{1}{\eta_1} \frac{1+\beta^2}{\sigma} J_H + \beta E_y \eta_2 - \beta UB\eta_3 & \end{aligned} \tag{5}$$

Here the plasma properties $\sigma$ and $\beta$ are the freestream values. The quantity $\eta_n$ is unity when no boundary layers are present ($\delta_m/H \ll 1$) and always larger than $(1 - 2\delta_m/H)$ since the $f_n$ factors are positive. These $f_n$ factors may be viewed as ratios of four boundary-layer thicknesses to the larger of either the thermal or viscous boundary layers. When these ratios are large $[(\delta_m/H)f \simeq 1]$ boundary layer shorting effects are important. We may expect the $f_n$'s to be large if the conductivity in the boundary layer is much larger than that of the freestream, since the other ratios in the $f_n$ integrals are of order unity or less.

The voltage current characteristic and the Hall field expressions in Eq. (5) are written so that both the Hall generator and the Faraday generator may be analyzed. In the case of the Hall generator, $E_y$ is set equal to zero if electrode drops are neglected. We will henceforth restrict our attention to the segmented electrode Faraday generator for which $J_H = 0$. Some interesting operating points for this generator may be obtained.

1) open circuit (o.c.) voltage

$$(E_y/UB)_{o.c.} = (\eta_1\eta_4 + \eta_2\eta_3\beta^2)/(\eta_1^2 + \eta_2^2\beta^2) \tag{6a}$$

Note that $\eta_3$ and $\eta_4$ contain the ratio $u/U$ and $\eta_1$ and $\eta_2$ do not. Thus it follows that $E_y/UB < 1$ at open circuit.

2) short circuit (s.c.) current

$$(J/\sigma UB)_{s.c.} = -(1/\eta_1)(\eta_1\eta_4 + \eta_2\eta_3\beta^2)/(1 + \beta^2) \tag{6b}$$

This quantity is slightly less than unity by a similar argument.

3) Hall recovery

$$\xi = E_x/\beta(E_y - UB) = (\eta_3 - \eta_2 k)/\eta_1(1 - k)$$

where $k$ is the load factor and is defined by $k = E_y/UB$, which becomes in the open circuit case where $k = (E_y/UB)_{o.c.}$

$$\xi_{o.c.} = \frac{\eta_1\eta_3 - \eta_2\eta_4}{\eta_1(\eta_1 - \eta_4) + \beta^2\eta_2(\eta_2 - \eta_3)} \tag{6c}$$

and at the short circuit condition $\xi_{s.c.} = \eta_3/\eta_1$ which is less than unity.

If the quantities $\eta_n$, are known, these relations yield the over-all performance of the generator. Of course, their calculation requires a detailed knowledge of the boundary-layer behavior.

In order to assess the effect of the various quantities which enter the $\eta$ factors it is useful to simplify these expressions. The first simplification we can make is to ignore variations in the Hall parameter through the boundary layer. This assumption is justifiable when the macroscopic collision cross section of the plasma is not sensitive to changes in electron temperature. The effect of the velocity ratio $u/U$ may be investigated by considering two limiting cases: 1) $u/U = 1$ which means that there exists only a temperature nonuniformity in the boundary layer and the region of the velocity defect is very close to the wall. 2) $u/U = 0$ which means that the region of large conductivity variation is close to the wall where the velocity is low.

### Temperature Nonuniformity with $u/U = 1$

In this case the $\eta$ factors are all identical and can be written

$$\eta = 1 + \psi \tag{7}$$

where $\psi = 2(\delta_m/H)(f - 1)$. The quantity $\psi$ is zero when the conductivity is uniform, slightly negative ($\psi = -2\delta_m/H$) when the boundary layer is nonconducting, and positive when the boundary layer is shorting. One can write the performance relations for this case as follows:

$$\begin{aligned} J/\sigma UB &= (k-1)(1+\psi) \\ E_x/UB &= \beta(k-1),\ \xi = 1 \end{aligned} \tag{8}$$

The current densities in the freestream are

$$j_x/\sigma UB = 0; j_y/\sigma UB = k - 1 \tag{9}$$

We note that the net output current is greater than the freestream current. This is consistent with the assumption that $u/U = 1$ since here the boundary layer also serves as a generator, adding to the output current. Since the Hall recovery is unity, which implies that nonequilibrium ionization is possible, we note that the presence of only a temperature or conductivity nonuniformity is not serious in degrading the MHD generator performance. We expect also that a temperature nonuniformity alone, without a velocity defect, is difficult to realize physically. Viscous and heat conduction effects are characterized by the same length scale since the Prandtl number is of order unity for gases. It appears that the limit $u/U = 0$ is physically more realistic.

### Temperature Nonuniformity with $u/U = 0$

With the assumptions of the preceding section we obtain for the $\eta$ factors

$$\eta_1 = \eta_2 = (1 + \psi)$$

and (10)

$$\eta_3 = \eta_4 = 1 \text{ if } 2\delta_m/H \ll 1$$

The voltage-current characteristic then becomes

$$J/\sigma UB = k(1 + \psi) - 1 \tag{11}$$

Hall potential

$$E_x/UB = \beta[k - 1/(1 + \psi)] \tag{12a}$$

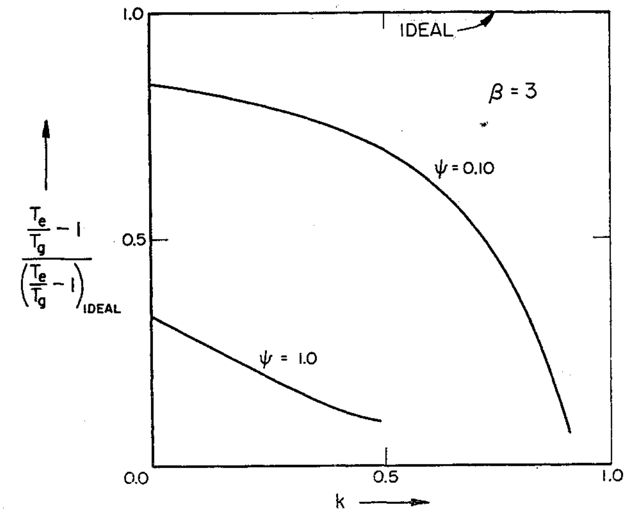


**Fig. 2 Electron temperature rise vs load factor for insulator wall boundary-layer shorting with constant shorting parameter $\psi$.**

Hall recovery

$$\xi = [k - 1/(1 + \psi)]/(k - 1) \tag{12b}$$

Freestream currents

$$\begin{aligned} j_x/\sigma UB &= [\beta/(1 + \beta^2)]\psi/(1 + \psi) \\ j_y/\sigma UB &= k - 1 + [\beta^2/(1 + \beta^2)]\psi/(1 + \psi) \end{aligned} \tag{13}$$

Note that the behavior approaches that of an ideal generator when $\psi \rightarrow 0$. Choosing a pessimistic value of $\psi \simeq 1$ we see that the open circuit voltage is 50% of the ideal value. At short circuit ($k = 0$), the Hall recovery is 50%, which indicates that the conductivity and hence the current density to the load may also be seriously affected.

### Generator Behavior with $u/U = 0$

The generator performance may be assessed by computing the variation of the electron temperature variation. The electron temperature is a direct measure of the gas conductivity if the heavy gas temperature and gas choice are specified. The electron temperature elevation is related to the Joule dissipation through the electron energy equation[5]

$$j^2/\sigma = (\delta m_e/m_a)n_e\nu_c\tfrac{3}{2}k(T_e - T_g) \tag{14}$$

where radiation and heat conduction losses have been neglected. Here $\delta$ is the collision energy loss parameter. This equation may be rewritten for a generator as follows:

$$T_e/T_g - 1 = (2\gamma/3\delta)\beta^2M^2[j^2/(\sigma UB)^2] \tag{14a}$$

where the term in brackets is the nondimensional joule dissipation. In comparing two flows with the same gasdynamic conditions we may say that

$$\frac{T_e/T_g - 1}{(T_e/T_g - 1)_{\text{ideal}}} = \frac{(j/\sigma UB)^2}{(j/\sigma UB)^2_{\text{ideal}}} \tag{15}$$

where the terms in the numerator are those including a particular loss. For the insulator wall boundary-layer shorting we have

$$\frac{j^2}{(\sigma UB)^2} = \frac{j_x^2 + j_y^2}{(\sigma UB)^2} = (1 - k)^2 + \frac{\beta^2}{1 + \beta^2}\frac{\psi}{1 + \psi}\left[2(k - 1) + \frac{\psi}{1 + \psi}\right] \tag{16}$$

and for the ideal case, where the shorting parameter $\psi = 0$, the corresponding term may be easily computed. Forming the ratio of the two electron temperature parameters we have

$$\frac{T_e/T_g - 1}{(T_e/T_g - 1)_{\text{ideal}}} = 1 - \frac{\beta^2}{1 + \beta^2} \times \left[2\left(\frac{\psi}{1 + \psi}\cdot\frac{1}{1 - k}\right) - \left(\frac{\psi}{1 + \psi}\frac{1}{1 - k}\right)^2\right] \tag{17}$$

This ratio is shown in Fig. 2 for $\beta = 3$ and values of the shorting parameter $\psi = 0.1$ and 1.0. For a specified degree of shorting the electron temperature is depressed and therefore so is the conductivity. Even for $\psi = 0.1$ this effect is serious because of the exponential variation of conductivity with temperature. We note the effect of Hall field ($E_x$) shorting near $k = 0$ and transverse field ($E_y$) shorting where the load factor approaches its open circuit value (which is less than 1).

Another measure of generator performance is the efficiency. Let us define the turbine efficiency as the ratio of power output to the enthalpy flux change. This may be broken into the following ratios:

$$\eta = \frac{\text{power output}}{\Delta\text{ enthalpy flux}} = \frac{\text{power output}}{\text{freestream power output}}\cdot\frac{\text{freestream power output}}{\Delta\text{ enthalpy flux}} \tag{18}$$

The second ratio is the polytropic efficiency of the generator which is equal to the load factor, $k$, if compressibility, friction, and heat-transfer effects are neglected.[6] The first ratio is

$$\frac{JE_y}{j_xE_x + j_yE_y} = \frac{k[1 - (1 + \psi)k]}{k(1 - k) + [\beta^2/(1 + \beta^2)][\psi/(1 + \psi)][1/(1 + \psi) - 2k]} \tag{19}$$

with Eqs. (11–13) the efficiency becomes

$$\eta = \frac{k^2[1 - (1 + \psi)k]}{k(1 - k) + [\beta^2/(1 + \beta^2)][\psi/(1 + \psi)][1/(1 + \psi) - 2k]} \tag{20}$$

This quantity is shown in Fig. 3. We note that the curves with shorting follow slightly below the ideal curve ($\psi = 0$, $\eta = k$) to a maximum value just short of open circuit. Except for the impossibility of operating near open circuit, this shorting effect seems small, but it must be remembered that shorting implies a lower conductivity hence a longer interac-

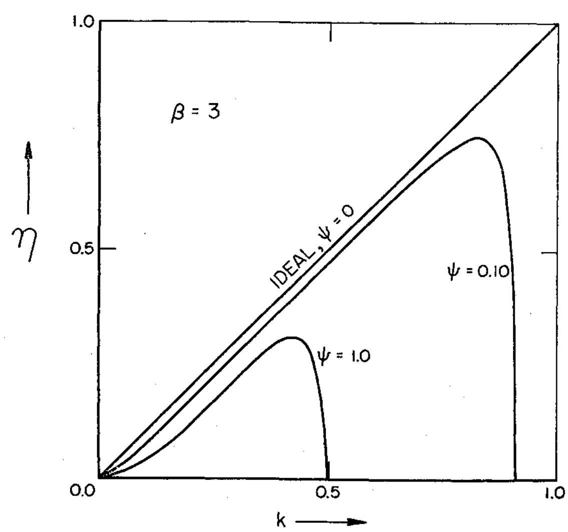


**Fig. 3 Efficiency vs load factor for insulator wall boundary layer shorting with constant values of the shorting parameter, $\psi$.**

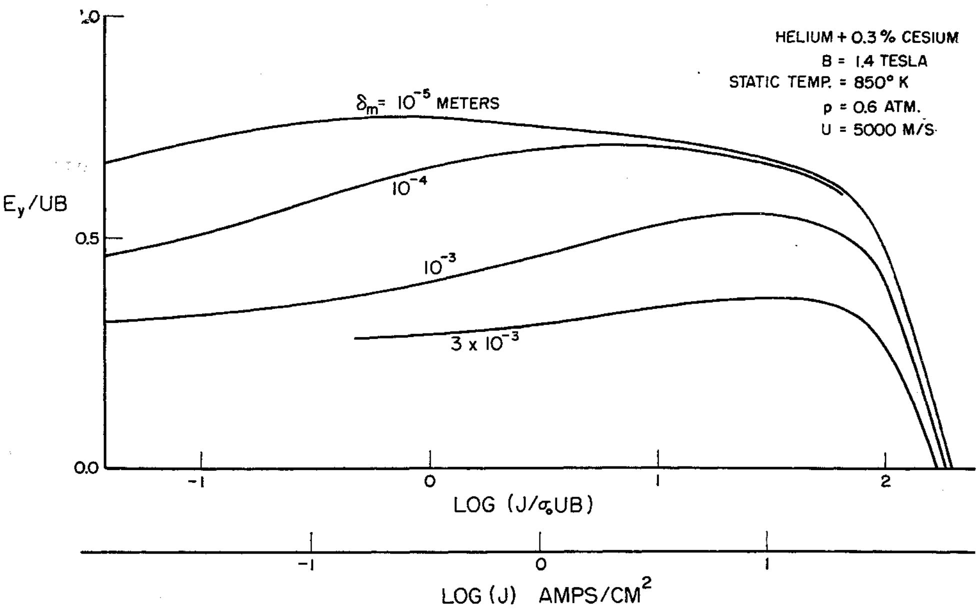


**Fig. 4 Voltage-current characteristic for a nonequilibrium generator shorted through a boundary layer of thickness $\delta_m$ and with a gas temperature of 2000°K. $\sigma_o$ = conductivity frozen at stagnation conditions $\simeq$ 6 mhos/m.**

tion length and consequently larger wall losses such as friction and heat transfer.

So far we have exploited the parameter $\psi$ to gain an understanding of the boundary-layer shorting effects. In a combustion driven generator where the local static temperature determines the conductivity in the boundary layer, $\psi$ may be calculated from a knowledge of the temperature and velocity distributions. In this case, $\psi$ is not expected to vary strongly with the load condition, so that $\psi$ is a constant for given flow conditions. In the case of nonequilibrium plasmas where the conductivity is a strong function of the local electric field, $\psi$ depends on the load condition and the flow conditions.

The parameter $\psi$, $\simeq 2(\delta_m/H)\sigma'/\sigma$, may be used to determine the generator size, $H$, required to make the boundary-layer shorting negligible. For the combustion driven generator the degree of ionization, and hence the conductivity, depends only on the static temperature if the pressure is fixed. We may say that for small degrees of seed ionization, the Saha equation gives (approximately)

$$\sigma \sim e^{-\epsilon_i/2kT}$$

and therefore

$$\sigma'/\sigma = \exp(-\epsilon_i/2kT' + \epsilon_i/2kT)$$

where $\epsilon_i$ = ionization potential of the seed; $k$ = Boltzmann's constant; $T$ = static temperature of the freestream; $T'$ = static temperature of the boundary layer, which we may take to be the recovery temperature or the stagnation temperature, $T_0$, of the fluid. We may then say

$$T'/T \simeq T_0/T = 1 + [(\gamma - 1)/2]M^2$$

where $M$ is the flow Mach number.

The preceding may be combined to yield

$$\sigma'/\sigma \cong \exp + [\epsilon_i/2kT_0][(\gamma - 1)/2]M^2$$

The generator operating characteristics [Eqs. (11) and (12) and Fig. 3] dictate a maximum acceptable value of $\psi$. A corresponding ratio $\delta_m/H$ is then specified for a given set of fluid parameters

$$\delta_m/H = (\psi/2)\exp - (\epsilon_i/2kT_0)[(\gamma - 1)/2]M^2$$

For example, if $\psi = 0.1$ in a typical combustion driven generator[7] where $\epsilon_i = 4.5$ ev, $T_0 = 3300$°K, $M = 1.4$, and $\gamma = 1.2$, this relation yields $\delta_m/H = 0.01$.

In order to investigate the voltage-current characteristics of a nonequilibrium MHD generator with insulator wall shorting effects, numerical methods must be employed. This is necessary because of the algebraic complexity of the Saha equation and the expression for the Coulomb collision cross section.

**Numerical Example**

As an illustration of the resulting characteristics, a mixture of helium and cesium is chosen as a working fluid. The Hall parameter is again fixed and assumed to be the same in the insulator wall boundary layer as in the freestream. This assumption, although incorrect when considering coulomb effects in the conductivity, reduces the number of $\eta$ factors to the important ones. Setting $\beta' = \beta$ should not seriously affect the physical result because the variations of the conductivity ratio and the velocity ratio in the $\eta$ factors dominate the variation of the Hall parameter. The effective boundary-layer thickness is treated parametrically, using the channel height, $H$, as a fixed constant.

Inputs to the calculation are the following: 1) thermodynamic variables: static freestream and wall layer temperatures, pressure; 2) seed fraction of cesium; 3) Hall parameter, the product $[2\gamma/3\delta]M^2$; 4) boundary-layer thickness, $\delta_m$.

The electron temperatures in the freestream and in the boundary layer ($T_{e\infty}$ and $T_{ew}$) can be computed from the electron energy equation (neglecting terms due to radiation and heat conduction) if the electric fields $E_y$ and $E_x$ are known;

$$T_{e\infty} = T_g\left\{1 + \frac{2\gamma}{3\delta}M^2\frac{\beta^2}{1+\beta^2}[(1-k)^2 + h^2]\right\} \tag{21a}$$

here $h = E_x/UB$ and

$$T_{ew} = T_g\left\{\frac{T_w}{T_g} + \frac{2\gamma}{3\delta}M^2\frac{\beta^2}{(1+\beta^2)}[(k)^2 + h^2]\right\} \tag{21b}$$

This permits the calculation of the conductivities $\sigma$ and $\sigma'$ and therefore the parameter $\psi$. Equation (12) then gives $E_x/UB$ for a given $E_y/UB$.

This system of equations must be solved by iteration and gives as a result the voltage-current characteristic and the Hall recovery for the given conditions. These are shown in Figs. 4–7 for a generator with the properties shown in Table 1. This example corresponds to conditions existing in an experimental investigation carried out by the author.[8]

The V-I characteristic is plotted as $E_y/UB$ vs $\log J/\sigma_0 UB$ where $\sigma_0$ is the conductivity corresponding to the stagnation conditions. Labeled "ideal" in the plots is the case $\delta_m = 10^{-9}m$, $T_w = 600$°K for which shorting is negligible. The characteristics show that there are three operating points available for some value of $E_y$. The first point characterized

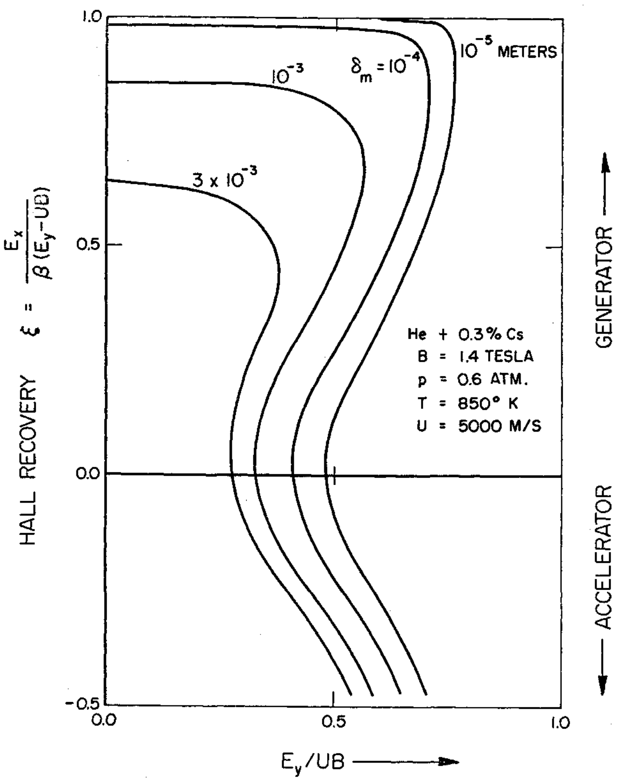


**Fig. 5 Hall recovery for a nonequilibrium generator shorted through a boundary layer of thickness $\delta_m$ and with a gas temperature of 2000°K.**

by a large net output current and a large Hall recovery is a normal operating point. The second with a smaller net output current and small Hall recovery is unstable except perhaps $J = 0$. A third point, with $J$ and $E_x$ reversed, corresponds to an accelerator mode of operation. (This point is not shown on the V-I diagram which has a logarithmic abscissa.) We note that this triple valued behavior is much more predominant when the wall layer is hot than when it is cold. The figures also show that near short circuit neither the hot wall nor the cold wall has much effect on the output current. This is because at the high electron temperatures of that condition the conductivities are relatively insensitive to changes in the electron temperature which means that the conductivities are nearly equal and $\psi$ is therefore small.

**Table 1 Summary of physical variables used in the numerical example**

| | |
|---|---|
| Gas mixture | Helium + 0.003 Cesium |
| Mach number | $M = 2$ |
| Hall parameter | $\beta = 4$ |
| Average wall layer temperature | $T_w = 600, 2000°K$ |
| Stagnation temperature | $T_0 = 2000°K$ |
| Stagnation pressure | $P_0 = 5$ atm |
| Energy loss parameter | $\delta = 2$ |
| Specific heat ratio | $\gamma = 5/3$ |
| Channel height | $H = 3.75$ cm |

Near open circuit, however, it does not require much joule heating in the case of the hot wall to make the boundary layer a very effective short. We note that the open circuit voltage is lower than the maximum voltage in both cases.

These generator characteristics should be useful in identifying shorting through the insulator wall boundary layer of a nonequilibrium MHD generator, although the generator for which these calculations were performed was probably not affected by this phenomenon.[2]

Since $\delta_m$ is the boundary-layer height with an appreciable velocity defect, we may relate $\delta_m$ to the real boundary-layer thickness by specifying the velocity ratio at the edge of the slow portion of the boundary layer. Taking this to be $u/U = 0.5$ for a turbulent boundary layer with a $\frac{1}{7}$ power profile we obtain

$$\delta_m/\delta = (0.5)^7 \simeq 0.01$$

We have seen from the examples that generators with $\delta_m/H$ greater than $10^{-3}$–$10^{-4}$ may suffer performance degradation from shorting through the insulator wall boundary layer. The channel height, $H$, should therefore be 10–100 times larger than the boundary-layer thickness, $\delta$.

### Scalar Wall Conduction Shorting

The general characteristic expressions involving $\eta_n$ [Eq. (5)] may be specialized to assess the shorting behavior of an insulator wall with finite conductivity. The properties of the wall are characterized by $u = 0$ (stationary wall) and $\beta = 0$ because the Hall effect is very small in materials other than gases. The factors $f_2$, $f_3$, and $f_4$ become zero and hence $\eta_2$, $\eta_3$, and $\eta_4$ are equal to unity. The voltage-current characteristics and the Hall potential may be written in terms of a new shorting parameter $\Phi = 2\bar{\sigma}_w/\sigma H$ where $\bar{\sigma}_w$ is the surface

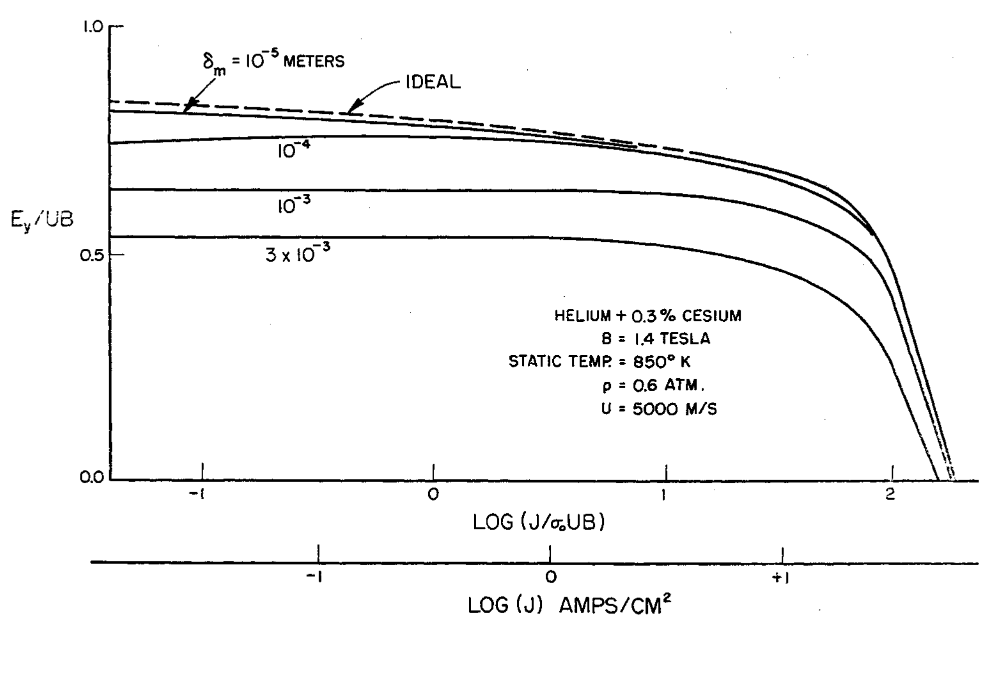


**Fig. 6 Voltage-current characteristic for a nonequilibrium generator shorted through a boundary layer of thickness $\delta_m$ and with a gas temperature of 600°K. $\sigma_o$ = conductivity frozen at stagnation conditions $\simeq$ 6 mhos/m.**

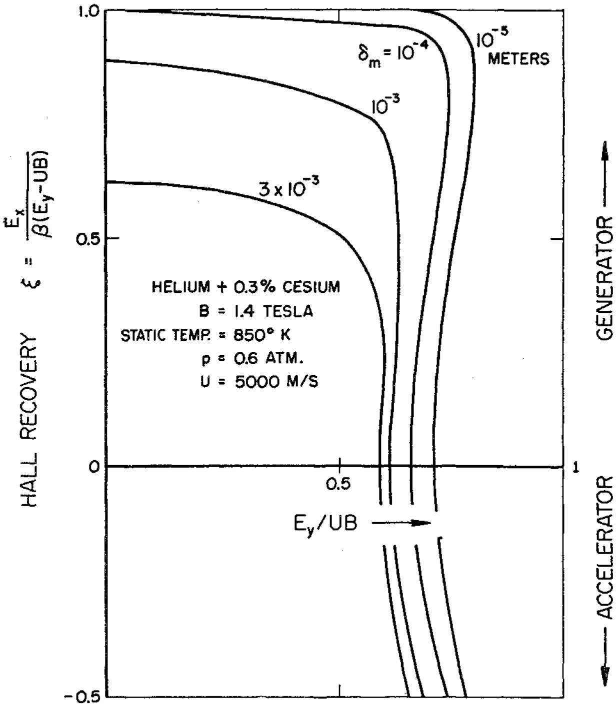


**Fig. 7 Hall recovery for a nonequilibrium generator shorted through a boundary layer of thickness $\delta_m$ and with a gas temperature of 600°K.**

conductivity of the insulator wall measured in mho;

$$\frac{E_y}{UB}\,\frac{(1+\Phi)^2+\beta^2}{1+\beta^2} - \frac{(1+\Phi)+\beta^2}{1+\beta^2} = \frac{J}{\sigma UB}\,(1+\Phi)$$

$$E_x/UB = \beta[(E_y/UB) - 1]/(1+\Phi)$$

Analogously to the preceding, these two equations with the four measured quantities ($E_y/UB$, $E_x/UB$, $J/\sigma UB$, $\beta$) can be used to check whether a single parameter $\Phi$ exists. If it does, its existence implies that the postulated shorting mechanism is one which may be responsible for the observed behavior. Confidence in this conclusion may be increased by noting whether the above equations are also satisfied at other load conditions. The value of $\Phi$ is independent of the load condition in the case of a combustion driven generator and varies ($1/\sigma$) with the electric field if the conductivity is obtained by nonequilibrium ionization. In either case $\bar{\sigma}_w$ should be independent of local electric field strengths.

## 3. Conclusions

The equations governing the behavior of a segmented electrode Faraday MHD generator affected by insulator boundary-layer shorting are derived. It is shown that four characteristic boundary-layer thicknesses exist and these are combined variations of the conductivity, the flow velocity, and the Hall parameter. With some simplifications these equations are investigated in detail. They show that shorting through the boundary layer may be serious if $H/\delta$ is less than ~10–100: operation is difficult near open circuit where the efficiency is high. Near short circuit the Hall field is effectively shorted preventing the attainment of a high electron temperature. The consequently low conductivity and long interaction length result in larger viscous and heat-transfer losses.

This theoretical model may be used to identify shorting through the insulator wall boundary layer by evaluation of the parameter $\psi$ for experimental conditions of interest. By appropriate specialization of the equations, relations are obtained which may be useful in detecting shorting through the insulator walls with finite conductivity.

## References

[1] Kerrebrock, J. L. et al., "A Large Nonequilibrium M.H.D. Generator," *Proceedings of the International Symposium of M.H.D. Electrical Power Generation*, Salzburg, Paper SM 74/163, June 1966.

[2] Decher, R. and Kerrebrock, J. L., "Electrode Wall-end Loop Shorting in a Nonequilibrium M.H.D. Generator," *Proceedings of the Ninth Symposium on Engineering Aspects of Magnetohydrodynamics*, Tullahoma, Tenn., April 1968, pp. 142–155.

[3] Hoffman, M. A., "Nonequilibrium MHD-Generator Losses Due to Wall and Insulator Boundary-Layer Leakages," *AIAA Journal*, Vol. 6, No. 9, Sept. 1968, pp. 1667–1673.

[4] Decher, R., "Effect of Boundary Layers on the Characteristics of a Nonequilibrium Generator," Quarterly Progress Report of the Research Laboratory of Electronics, Dec. 1966, M.I.T.

[5] Kerrebrock, J. L., "Nonequilibrium Ionization Due to Electron Heating: I. Theory," *AIAA Journal*, Vol. 2, No. 6, June 1964, pp. 1072–1080.

[6] Sutton, G. W. and Sherman, A., *Engineering Magnetohydrodynamics*, McGraw-Hill, New York, 1965.

[7] Louis, J. F., Gal, G., and Blackburn, P. R., "Detailed Theoretical and Experimental Study on a Large MHD Generator," *AIAA Journal*, Vol. 3, No. 8, Aug. 1965, pp. 1482–1490.

[8] Decher, R., "Experimental Investigation of a Nonequilibrium Magnetohydrodynamic Generator," Ph.D. thesis, Feb. 1968, Dept. of Aeronautics and Astronautics, M.I.T.